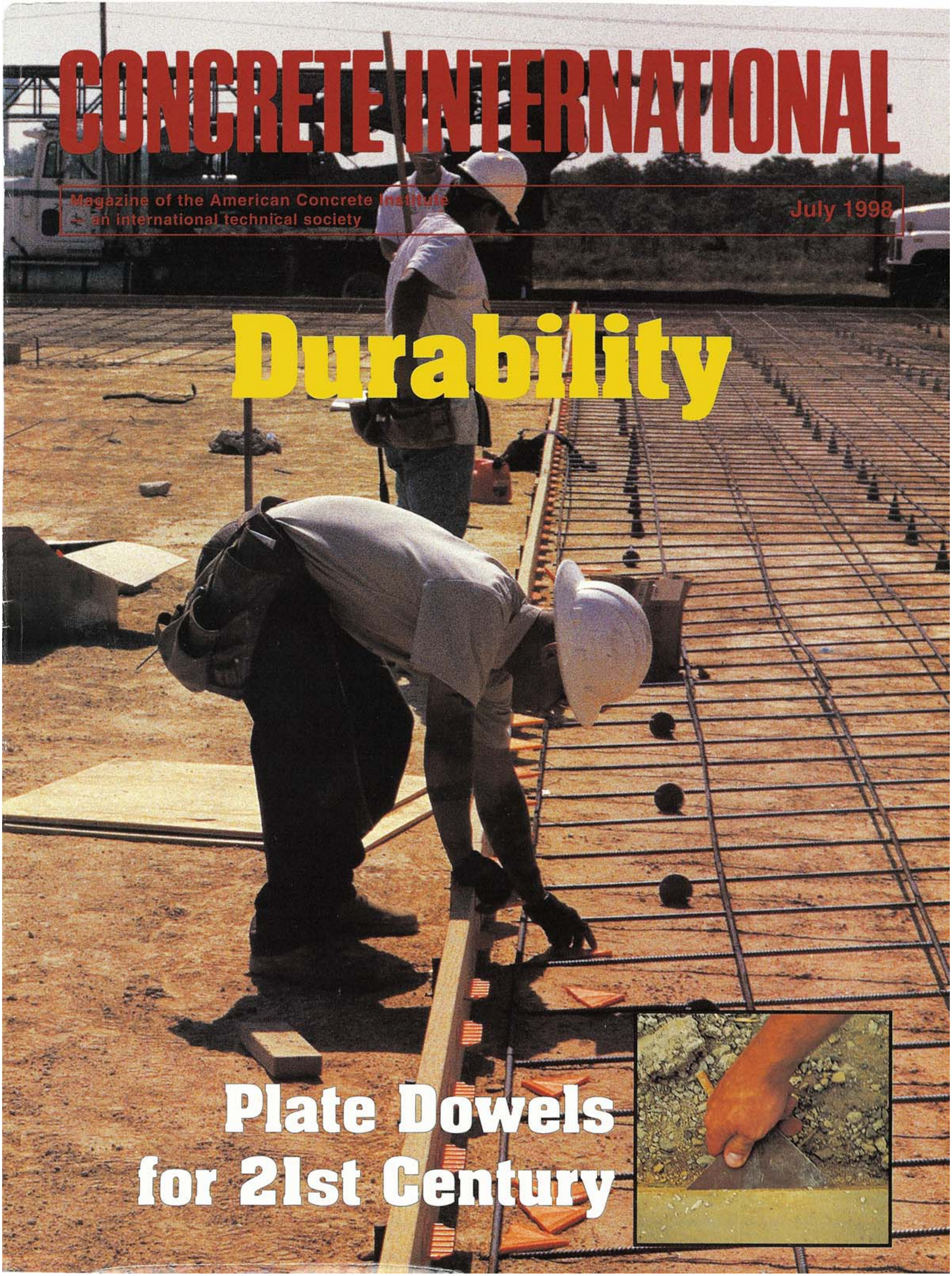
CONCRETE INTERNATIONAL

Magazine of the American Concrete Institute
— an international technical society

July 1998



Durability

Plate Dowels for 21st Century

Plate Dowels for Slabs on Ground

by Wayne W. Walker and Jerry A. Holland

Using plate dowels in slabs on ground for shear load transfer at the joints offer many advantages over the traditional round dowels. By using plate dowels, compressible material or a pocket former can be easily added to the sides of the plate dowels to accommodate the slab's horizontal movement parallel to the joint.

Allowing the slab to move unrestrained horizontally will help minimize the size and number of restraint cracks. This ability to accommodate the horizontal differential movement is especially important for slabs that have two directional doweling and for slabs with long joint spacings and significant movements, such as those incorporating post-tensioning or shrinkage-cornpensating concrete.

A plate dowel is a more efficient use of material and more cost effective than the traditional round dowel bars. Two types of plate dowels have been evaluated: rectangular plate dowels to be used in contraction (control) joints and diamond plate dowels to be used in construction joints. Fig. 1(a)

Worker installing diamond plate dowel pocket formers on base supports, which were previously attached to form. Note no form penetrations are required. Note short dowel length and long spacing as compared to typical dowels.

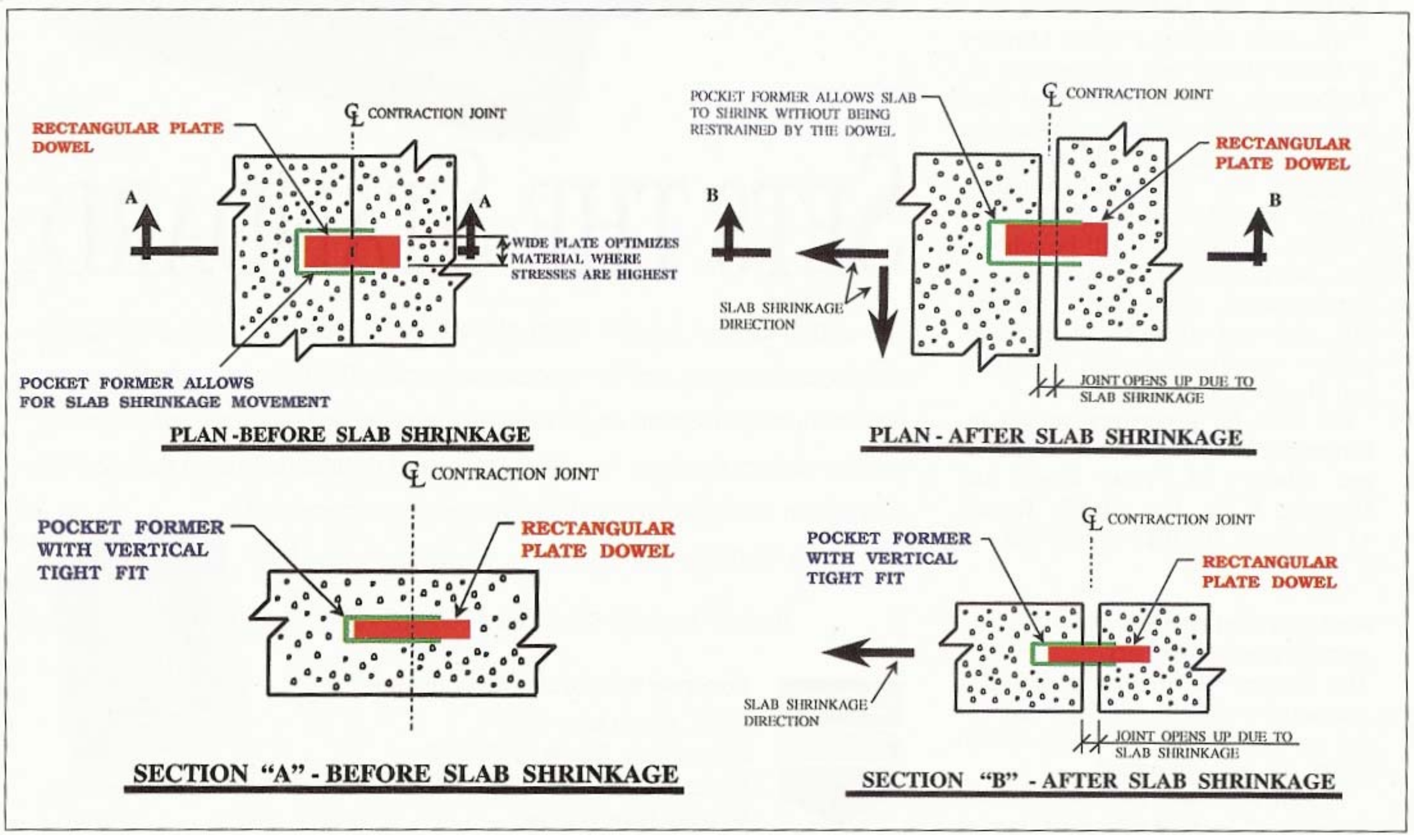


Fig. 1(a) - Rectangular plate dowel before slab shrinkage.

Fig. 1(b) - Rectangular plate dowel after slab shrinkage.

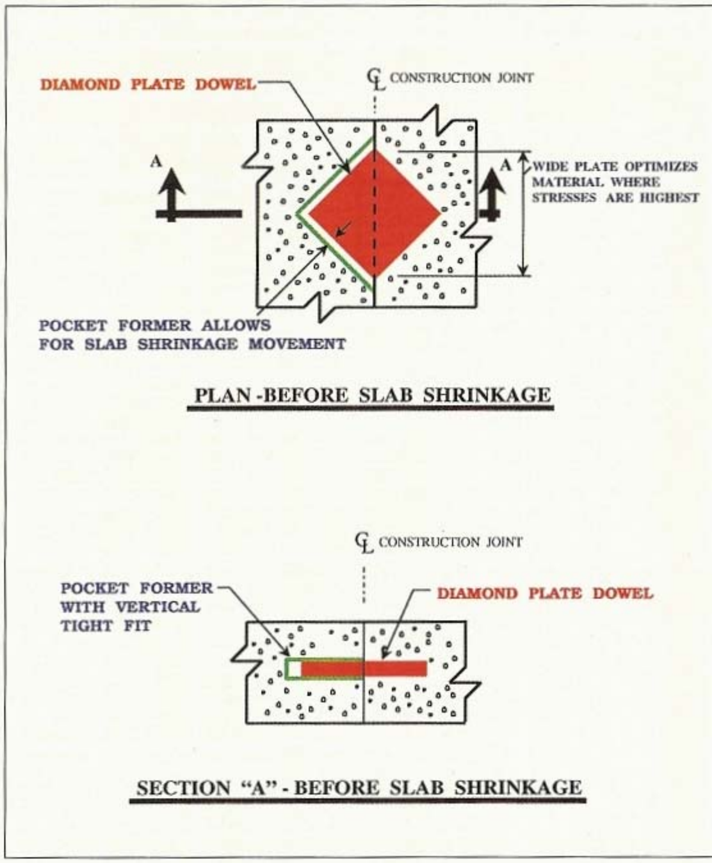


Fig. 2(a) - Diamond plate dowel before slab shrinkage.

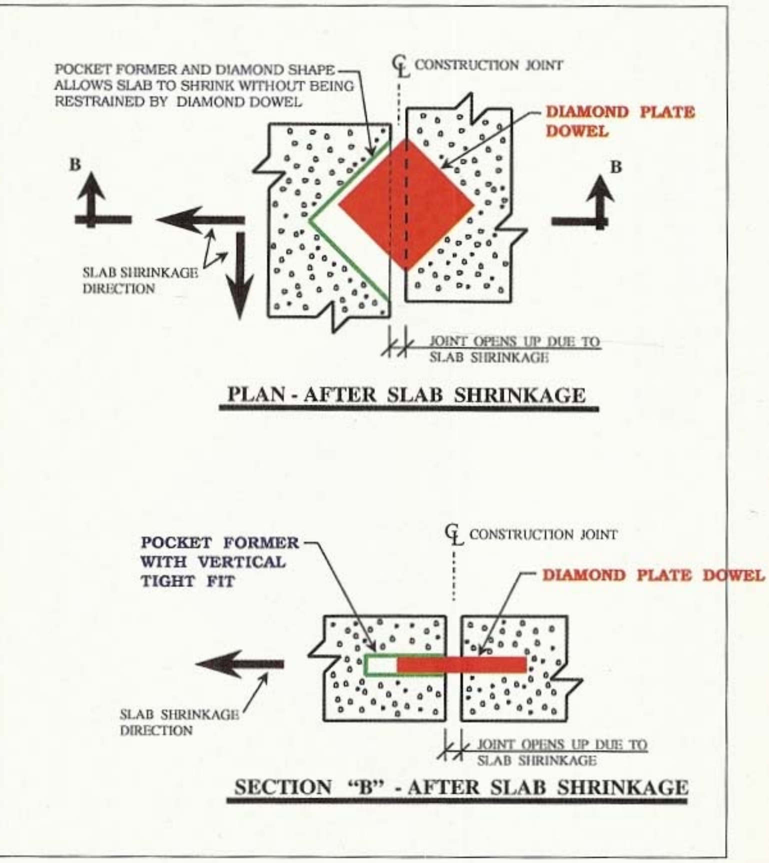


Fig. 2(b) - Diamond plate dowel after slab shrinkage.

and 2(a) show the rectangular and diamond plate dowels installed in the slab before any movement has taken place. Fig. 1(b) and 2(b) show the dowels after movement has taken place: the advantages of allowing the slab to move unrestrained can be clearly seen. The diamond plate dowel offers the most advantages because the plate material is in the optimum location and the shape will allow the slab to move horizontally in all directions without restraint.

Because plate dowels have not been used in this manner previously, there are no industry recommendations for the size and spacing. The authors provide recommendations for plate dowels such that they will have the equivalent stress and serviceability performance as the round dowel bars spaced at 12 in. (300 mm) on center that are presently recommended in ACI 302.1 R, "Guide for Concrete Floor and Slab Construction."[1] Equivalent plate dowel spacings for round dowel bars spaced at 18 in. (460 mm) on center are also provided because this spacing is commonly used.

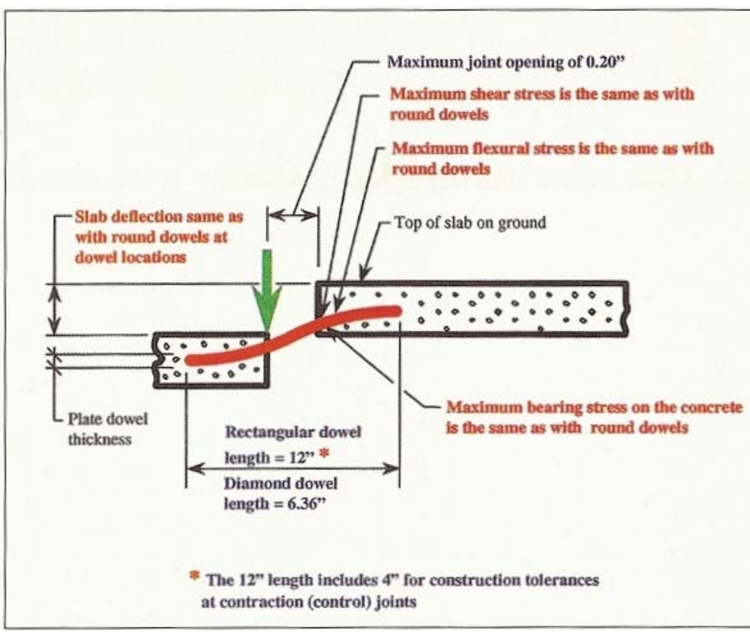


Fig. 3 - Plate dowel performance requirements.

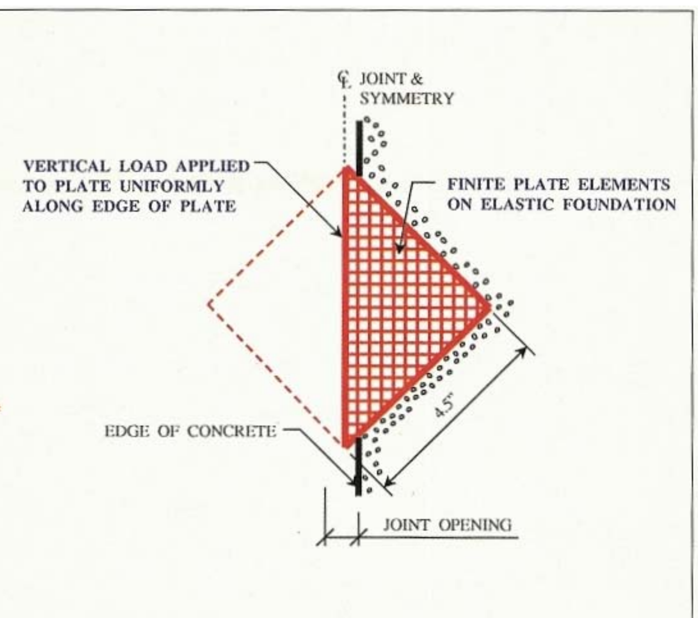


Fig. 4 - Computer model for diamond plate dowel.

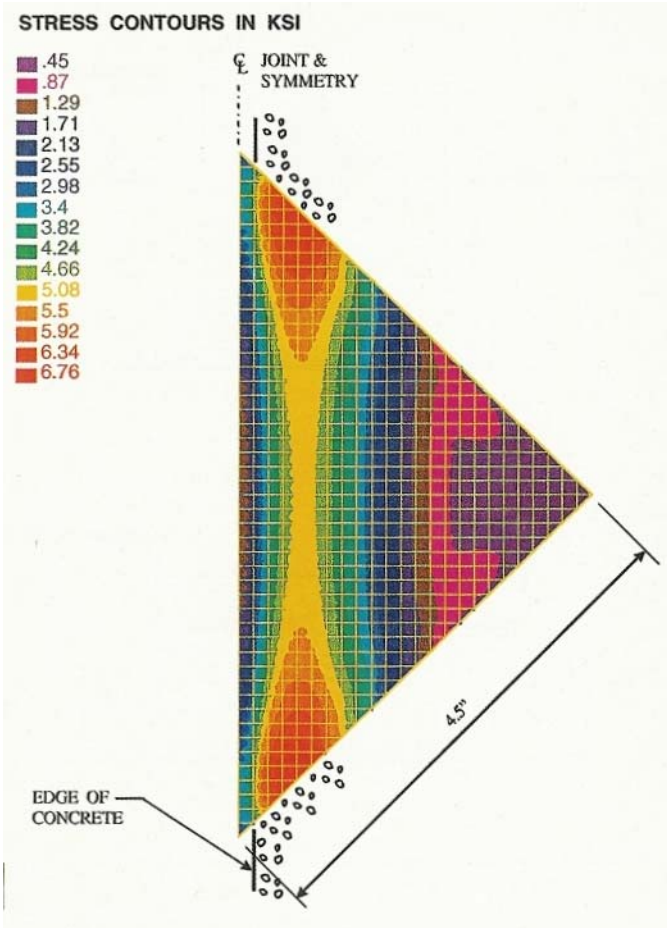


Fig. 5 - Flexural stresses for a 1/4 in. thick diamond plate dowel with a one kip load applied to the edge.

The performance criteria used was such that the plate dowels would have the same stiffness (vertical deflection between slabs at the dowel locations, see Fig. 3), bearing stress on the concrete, and dowel flexural stress and shear stress as the round dowels presently recommended. The recommendations are presented in a tabular form so that several options between plate dowel sizes and spacings can be evaluated by the slab's designer.

Analytical approach

Extensive computer programs were developed to analyze the plate dowels. The rectangular plate dowel was modeled as a beam on an elastic foundation,[3] and the equations that were used, along with their derivations, are in Appendix "A." (For Appendices, see pages 36 to 38 following this article).

An example problem showing a side by side comparison of the rectangular plate dowel performance with that of the round dowel can be found in Appendix "B." For the diamond plate dowel, a finite plate element on an elastic foundation analysis was used. Fig. 4 shows the computer model for the diamond plate dowel.

The concrete modulus of dowel support value is used for the elastic foundation analysis. Values given for the concrete modulus of dowel support range from 300 to 1500 kci (80 to 400 N/mm^3).[2,6] The authors varied the value between 300 and 1000 kci, and the results were changed very little; additionally, for values above 1000 kci (270 N/mm^3) the results were not significantly affected. Therefore, since the analysis is relatively insensitive to the value, the authors used a conservative value of 700 kci (190 N/mm^3) for the concrete modulus of dowel support.

A maximum joint opening size of 0.2 in. (5 mm) was used. This value should be sufficient for most typically used slab joint spacings and concrete mixes. For the unusual case where the joint may open wider, the spacings in the tables would be reduced. Since the spacing of the plate dowels are based on the equivalent performance of the round dowels at the same joint opening size, the reduction in the plate dowel spacing would be small.

Results

Using the analytical approach and values noted above, tables for sizes and spacings of rectangular and diamond plate dowels were developed. Table 1 gives the rectangular plate dowel spacing to match the round dowel performance.

TABLE 1

RECTANGULAR PLATE DOWEL SPACINGS TO MATCH ROUND DOWEL PERFORMANCE

Rectangular Plate Dowel Size	Center to Center Spacing for Rectangular Plate Dowels					
	3/4" Round Dowel Spacing		1" Round Dowel Spacing		1-1/4" Round Dowel Spacing	
Thickness X Width	12"	18"	12"	18"	12"	18"
3/8" X 0.75"		10"				
3/8" X 1.00"	9"	14"				
3/8" X 1.25"	12"	18"				
3/8" X 1.50"	14"	21"				
3/8" X 1.75"	16"	24"		12"		
3/8" X 2.00"	19"		9"	14"		
3/8" X 2.25"	21"		11"	16"		
3/8" X 2.50"	24"		12"	18"		11"
3/8" X 2.75"			13"	20"	8"	12"
3/8" X 3.00"			14"	22"	9"	13"
1/2" X 0.75"	10"	15"				
1/2" X 1.00"	13"	20"		11"		
1/2" X 1.25"	17"	24"	9"	13"		
1/2" X 1.50"	20"		11"	16"		
1/2" X 1.75"	23"		12"	19"		
1/2" X 2.00"	24"		14"	21"		13"
1/2" X 2.25"			16"	24"	9"	14"
1/2" X 2.50"			18"		10"	16"
1/2" X 2.75"			20"		12"	18"
1/2" X 3.00"			22"		13"	19"
5/8" X 0.75"	12"	18"		10"		
5/8" X 1.00"	16"	24"		14"		
5/8" X 1.25"	20"		12"	18"		11"
5/8" X 1.50"	24"		14"	21"		13"
5/8" X 1.75"			17"	24"		15"
5/8" X 2.00"			19"		11"	17"
5/8" X 2.25"			21"		13"	19"
5/8" X 2.50"			24"		14"	21"
5/8" X 2.75"					16"	24"
5/8" X 3.00"					17"	
3/4" X 0.75"	14"	20"		12"		
3/4" X 1.00"	18"	24"	11"	16"		11"
3/4" X 1.25"	23"		13"	20"		13"
3/4" X 1.50"	24"		16"	24"	11"	16"
3/4" X 1.75"			19"		12"	19"
3/4" X 2.00"			21"		14"	22"
3/4" X 2.25"			24"		16"	24"
3/4" X 2.50"					18"	
3/4" X 2.75"					20"	
3/4" X 3.00"					22"	

TABLE 2

DIAMOND PLATE DOWEL SPACINGS TO MATCH ROUND DOWEL PERFORMANCE

Diamond Plate Dowel Size	Center to Center Spacing for Diamond Plate Dowels					
	3/4" Round Dowel Spacing		1" Round Dowel Spacing		1-1/4" Round Dowel Spacing	
Thickness X SQ.Length	12"	18"	12"	18"	12"	18"
1/4" X 4.5"	18"	24"	11"	16"		10"
3/8" X 4.5"	24"	24"	17"	24"	11"	17"
1/2" X 4.5"	24"	24"	22"	24"	15"	22"
3/4" X 4.5"	24"	24"	24"	24"	20"	24"

For example, a 1/2 in. (13 mm) thick by 1 in. (25 mm) wide rectangular plate dowel spaced at 13 in. (330 mm) on center will provide the same slab performance as 3/4 in. (19 mm) round dowels spaced at 12 in. (300 mm) on center.

Table 2 gives the diamond plate dowel spacing to match the round dowel performance. For example, a 1/4 in. (6 mm) thick by 4.5 in. (114 mm) square diamond plate dowel spaced at 18 in. (460 mm) on center will provide the same slab performance as 3/4 in. round dowels spaced at 12 in. on center. The maximum spacing was limited to 24 in. (610 mm) to prevent the slab deflection between dowel locations from becoming significant. The performance values for the diamond plate and the round dowels can be found in Appendix "C." An example problem showing how the diamond plate spacing was determined can be found in Appendix "D."

A rectangular plate dowel embedment length of 4 in. (100 mm) past the joint was selected by the authors as being a practical value. The stresses reduce significantly beyond the first inch of the dowel past the joint, as can be seen in the graph from the example problem in Appendix "B." An embedment length longer than 4 in. would not provide any significant improvement in the dowel performance. The American Concrete Pavement Association has also recommended using similar short dowel embedment length values.[4,5] Since the rectangular plate dowels are to be used in contraction (control) joints, 4 in. should be added to the dowel length to account for construction tolerances. [4,5] This would give a total rectangular plate dowel length of 12 in.

A 4.5 in. square diamond plate dowel was also selected by the authors as being a practical value. As with the rectangular plate dowel, the stresses for the diamond plate dowel reduce significantly beyond the first inch of the dowel past the joint, as can be seen in Fig. 5. It can also be seen in Fig. 5 that only a small portion of the plate has the maximum stress. In developing the tables, the maximum peak stress and deflection values were conservatively used. The diamond plate dowel has significant reserve strength because this plate can redistribute the stresses if local yielding occurs. The diamond plate dowel is the optimum shape for a dowel. It is wide where the bearing, shear, and flexural stresses are the highest and is narrow where the stresses are reduced. The diamond shape also allows the slab to move horizontally without restraint when the slab's shrinkage opens the joint.

Conclusions

The authors provided recommendations for plate dowels such that they will have the equivalent stress and serviceability performance as the round dowel bars spaced at 12 in. on center that are presently recommended in ACI 302.1R "Guide for Concrete Floor and Slab Construction." Equivalent plate dowel spacings for round dowel bars spaced at 18 in. on center are also provided because this spacing is commonly used in the industry as well.

There are many advantages to using plate dowels in slabs on ground joints to transfer the shear forces. Some of the advantages are as follows:

1. Compressible material or a pocket former can be easily added to the sides of the plate dowels to accommodate the slab's horizontal movement parallel to the joint. Allowing the slab to move unrestrained horizontally will help minimize the size and number of restraint cracks.
2. A plate dowel is a more efficient use of material and more cost effective than the traditional round dowel bars. The diamond plate dowel is the optimum shape for a dowel.

Steel diamond plate dowels installed in plastic pocket formers. Pocket former base supports have been removed with form and will be reused. Note no oil or grease is required, thereby ensuring tighter fit but no restraint.

It is wide where the bearing, shear, and flexural stresses are the highest and is narrow where the stresses are reduced. The diamond shape also allows for the slab to move horizontally without restraint when the slab's shrinkage opens the joint.

Acknowledgments

The authors wish to thank Jim Saylor, Gordon Stallings, and Bob Anderson for their advice and recommendations.

References

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4. "Design and Construction of Joints for Concrete Highways," American Concrete Pavement Association, *Publication No.* TB010-01P, 1991.
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6. Syam S. Mannava, Thomas D. Bush, Jr., and Anant R. Kukreti, "Load-Deflection Behavior of Smooth Dowels," ACI Structural Journal, Nov.-Dec. 1999, pp. 891-898.

See following pages 36 to 38 for Appendices "A" through "D".

Note: This article has been updated since its first publication in July 1998

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APPENDIX "A"

EQUATION DEVELOPMENT FOR RECTANGULAR DOWEL PLATES

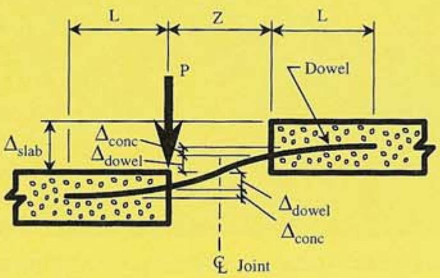


Fig. 1

From Fig.1 above, it can be seen that the deflection across the joint will be:

$$\Delta_{slab} = 2\cdot\left(\Delta_{conc} + \Delta_{dowel}\right)$$

The deflection of the dowel will consist of the shear deflection and the flexural deflection. Because the joint opening distance "z" is small, the shear deflection will be the predominate deflection of the dowel and will be added to the small flexural deflection of the dowel.

$$\Delta_{dowel} = \Delta_{flex} + \Delta_{shear} = \frac{P\cdot\left(\frac{z}{2}\right)^3}{3\cdot E\cdot I} + \frac{P\cdot\left(\frac{z}{2}\right)\cdot F}{G\cdot A} = \frac{P\cdot z^3}{24\cdot E\cdot I} + \frac{P\cdot z\cdot F}{2\cdot G\cdot A}$$

Where:

E = Modulus of elasticity for the steel dowel
I = Moment of inertia for the steel dowel
G= Shear modulus of elasticity for the steel dowel
A= Cross-sectional area for the steel dowel
F= Shear shape factor for the steel dowel

The deflection of the concrete (Δ_{conc}) can be estimated by assuming the dowel to be a beam on an elastic foundation. The equations for a beam on an elastic foundation for the model shown in FIG. 2 are from Ref. 3 .

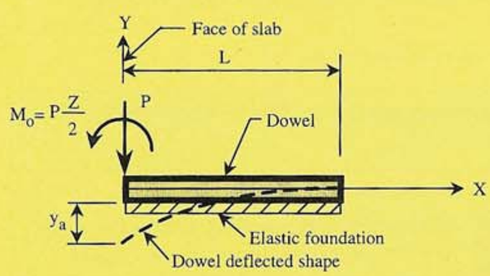


Fig. 2

For the concentrated load, the following equation was used:

$$y_{ap} = \frac{P}{2\cdot E\cdot I\cdot(\beta)^3}\cdot\frac{C_4\cdot C_1 - C_3\cdot C_2}{C_{11}}$$

For the applied moment, the following equation was used:

$$y_{am} = \frac{-P\cdot\frac{z}{2}}{2\cdot E\cdot I\cdot(\beta)^2}\cdot\frac{2\cdot C_3\cdot C_1 + (C_4)^2}{C_{11}}$$

Where :

$$\beta = \sqrt[4]{\frac{K\cdot b}{4\cdot E\cdot I}}$$

K = Concrete modulus of dowel support
b = Width of steel dowel

$$C_1 = \cosh(\beta\cdot L)\cdot\cos(\beta\cdot L)$$
$$C_2 = \cosh(\beta\cdot L)\cdot\sin(\beta\cdot L) + \sinh(\beta\cdot L)\cdot\cos(\beta\cdot L)$$
$$C_3 = \sinh(\beta\cdot L)\cdot\sin(\beta\cdot L)$$
$$C_4 = \cosh(\beta\cdot L)\cdot\sin(\beta\cdot L) - \sinh(\beta\cdot L)\cdot\cos(\beta\cdot L)$$
$$C_{11} = \sinh(\beta\cdot L)^2 - \sin(\beta\cdot L)^2$$

The total concrete deflection would be the summation of these two deflections.

$$\Delta_{conc} = y_{ap} + y_{am}$$

The total slab deflection would be:

$$\Delta_{slab} = 2\cdot\left[y_{ap} + y_{am} + \left(\frac{P\cdot z^3}{24\cdot E\cdot I} + \frac{P\cdot z\cdot F}{2\cdot G\cdot A}\right)\right]$$

The maximum bearing stress on the concrete would be given by:

$$\sigma_b = K\cdot\Delta_{conc}$$

The equation for the flexural stress in the dowel is:

$$f_b = \frac{M_p + M_m}{S}$$

Where:

S = Section modulus of the steel dowel
M_p = Moment in the steel dowel due to the concentrated load, and the equation is:

$$M_p = -y_{ap}\cdot 2\cdot E\cdot I\cdot(\beta)^2\cdot F_3 - \theta_{ap}\cdot E\cdot I\cdot\beta\cdot F_4 - \frac{P}{2\cdot\beta}\cdot F_2$$

M_m = Moment in the steel dowel due to the applied moment, and the equation is:

$$M_m = -y_{am}\cdot 2\cdot E\cdot I\cdot(\beta)^2\cdot F_3 - \theta_{am}\cdot E\cdot I\cdot\beta\cdot F_4 - \frac{P\cdot z}{2}\cdot F_1$$

Where:

$$F_1 = \cosh(\beta\cdot x)\cdot\cos(\beta\cdot x)$$
$$F_2 = \cosh(\beta\cdot x)\cdot\sin(\beta\cdot x) + \sinh(\beta\cdot x)\cdot\cos(\beta\cdot x)$$
$$F_3 = \sinh(\beta\cdot x)\cdot\sin(\beta\cdot x)$$
$$F_4 = \cosh(\beta\cdot x)\cdot\sin(\beta\cdot x) - \sinh(\beta\cdot x)\cdot\cos(\beta\cdot x)$$

$$\theta_{ap} = \frac{P}{2\cdot E\cdot I\cdot(\beta)^2}\cdot\frac{(C_2)^2 - 2\cdot C_3\cdot C_1}{C_{11}}$$

$$\theta_{am} = \frac{P\cdot\frac{z}{2}}{E\cdot I\cdot\beta}\cdot\frac{C_3\cdot C_4 + C_2\cdot C_1}{C_{11}}$$

The equation for the maximum shear stress in the dowel is:

$$f_v = \frac{P}{A}$$

APPENDIX "B"

EXAMPLE PROBLEM COMPARING RECTANGULAR PLATE DOWELS TO ROUND DOWELS

The following is an example problem showing that 1/2" thick X 1.0 " wide X 8" long rectangular plate dowel spaced at 13" on center provides the same performance as 3/4" round bars 18" long spaced at 12" on center. A 0.20 " joint opening distance is used.

Steel properties:

$E := 29000$ ksi $\quad G := 11154$ ksi

Rectangular plate dowel properties:

$F_{rec} := \frac{6}{5}$

$L_{rec} := 4$ in

$d_{rec} := 0.5$ in

$b_{rec} := 1.0$ in

$A_{rec} := b_{rec} \cdot d_{rec} \qquad A_{rec} = 0.5$ in²

$I_{rec} := \frac{b_{rec} \cdot d_{rec}^3}{12} \qquad I_{rec} = 0.0104$ in⁴

$S_{rec} := \frac{b_{rec} \cdot d_{rec}^2}{6} \qquad S_{rec} = 0.0417$ in³

Round dowel bar properties:

$F_{rou} := \frac{10}{9}$

$L_{rou} := 9$ in

$d_{rou} := 0.75$ in

$A_{rou} := \frac{\pi \cdot d_{rou}^2}{4} \qquad A_{rou} = 0.442$ in²

$I_{rou} := \frac{\pi \cdot d_{rou}^4}{64} \qquad I_{rou} = 0.0155$ in⁴

$S_{rou} := \frac{\pi \cdot d_{rou}^3}{32} \qquad S_{rou} = 0.0414$ in³

Let:
W_v = Unit uniform shear load along the joint of 1 kip/ft
S_{parec} = Spacing of the rectangular plate dowel
S_{parou} = Spacing of the round dowel

$W_v := \frac{1.0}{12}$ k/in

$S_{parec} := 13.0$ in $\qquad S_{parou} := 12$ in

$P_{rec} := S_{parec} \cdot W_v \quad P_{rec} = 1.083$ kips/dowel $\qquad P_{rou} := S_{parou} \cdot W_v \quad P_{rou} = 1$ kip/dowel

Check deflection across the joint

The joint opening distance is $z := 0.20$ in

Determine the flexural and shear deflection for the dowel:

$\Delta_{flexrec} := \frac{P_{rec} \cdot z^3}{24 \cdot E \cdot I_{rec}} \qquad \Delta_{flexrou} := \frac{P_{rou} \cdot z^3}{24 \cdot E \cdot I_{rou}}$

$\Delta_{flexrec} = 0.0000012$ in $\qquad \Delta_{flexrou} = 0.00000074$ in

$\Delta_{shearrec} := \frac{P_{rec} \cdot z \cdot F_{rec}}{2 \cdot G \cdot A_{rec}} \qquad \Delta_{shearrou} := \frac{P_{rou} \cdot z \cdot F_{rou}}{2 \cdot G \cdot A_{rou}}$

$\Delta_{shearrec} = 0.00002331$ in $\qquad \Delta_{shearrou} = 0.00002255$ in

Determine the concrete deflection:

Will use a concrete modulus of dowel support of $K := 700$ kci

$\beta_{rec} := \sqrt[4]{\frac{K \cdot b_{rec}}{4 \cdot E \cdot I_{rec}}} \quad \beta_{rec} = 0.8724$ 1/in $\qquad \beta_{rou} := \sqrt[4]{\frac{K \cdot d_{rou}}{4 \cdot E \cdot I_{rou}}} \quad \beta_{rou} = 0.7347$ 1/in

$C_{1rec} := \cosh(\beta_{rec} \cdot L_{rec}) \cdot \cos(\beta_{rec} \cdot L_{rec})$

$C_{2rec} := \cosh(\beta_{rec} \cdot L_{rec}) \cdot \sin(\beta_{rec} \cdot L_{rec}) + \sinh(\beta_{rec} \cdot L_{rec}) \cdot \cos(\beta_{rec} \cdot L_{rec})$

$C_{3rec} := \sinh(\beta_{rec} \cdot L_{rec}) \cdot \sin(\beta_{rec} \cdot L_{rec})$

$C_{4rec} := \cosh(\beta_{rec} \cdot L_{rec}) \cdot \sin(\beta_{rec} \cdot L_{rec}) - \sinh(\beta_{rec} \cdot L_{rec}) \cdot \cos(\beta_{rec} \cdot L_{rec})$

$C_{11rec} := \sinh(\beta_{rec} \cdot L_{rec})^2 - \sin(\beta_{rec} \cdot L_{rec})^2$

$C_{1rou} := \cosh(\beta_{rou} \cdot L_{rou}) \cdot \cos(\beta_{rou} \cdot L_{rou})$

$C_{2rou} := \cosh(\beta_{rou} \cdot L_{rou}) \cdot \sin(\beta_{rou} \cdot L_{rou}) + \sinh(\beta_{rou} \cdot L_{rou}) \cdot \cos(\beta_{rou} \cdot L_{rou})$

$C_{3rou} := \sinh(\beta_{rou} \cdot L_{rou}) \cdot \sin(\beta_{rou} \cdot L_{rou})$

$C_{4rou} := \cosh(\beta_{rou} \cdot L_{rou}) \cdot \sin(\beta_{rou} \cdot L_{rou}) - \sinh(\beta_{rou} \cdot L_{rou}) \cdot \cos(\beta_{rou} \cdot L_{rou})$

$C_{11rou} := \sinh(\beta_{rou} \cdot L_{rou})^2 - \sin(\beta_{rou} \cdot L_{rou})^2$

$$y_{aprec} := \frac{P_{rec}}{2 \cdot E \cdot I_{rec} \cdot (\beta_{rec})^3} \cdot \frac{C_{4rec} \cdot C_{1rec} - C_{3rec} \cdot C_{2rec}}{C_{11rec}}$$

$y_{aprec} = -0.0027$ in

$$y_{aprou} := \frac{P_{rou}}{2 \cdot E \cdot I_{rou} \cdot (\beta_{rou})^3} \cdot \frac{C_{4rou} \cdot C_{1rou} - C_{3rou} \cdot C_{2rou}}{C_{11rou}}$$

$y_{aprou} = -0.0028$ in

$$y_{amrec} := \frac{-P_{rec} \cdot \frac{z}{2}}{2 \cdot E \cdot I_{rec} \cdot (\beta_{rec})^2} \cdot \frac{2 \cdot C_{3rec} \cdot C_{1rec} + (C_{4rec})^2}{C_{11rec}}$$

$y_{amrec} = -0.00024$ in

$$y_{amrou} := \frac{-P_{rou} \cdot \frac{z}{2}}{2 \cdot E \cdot I_{rou} \cdot (\beta_{rou})^2} \cdot \frac{2 \cdot C_{3rou} \cdot C_{1rou} + (C_{4rou})^2}{C_{11rou}}$$

$y_{amrou} = -0.00021$ in

$\Delta_{concrec} := -(y_{aprec} + y_{amrec}) \qquad \Delta_{concrou} := -(y_{aprou} + y_{amrou})$

(note: the negative value is used so the value of the concrete deflection will be positive)

$\Delta_{concrec} = 0.00294$ in $\qquad \Delta_{concrou} = 0.003$ in

$\Delta_{slabrec} := 2 \cdot (\Delta_{concrec} + \Delta_{flexrec} + \Delta_{shearrec}) \qquad \Delta_{slabrou} := 2 \cdot (\Delta_{concrou} + \Delta_{flexrou} + \Delta_{shearrou})$

$\Delta_{slabrec} = 0.00593$ " is less than $\Delta_{slabrou} = 0.00606$ " O.K.

Determine the maximum bearing stress on the concrete

$\sigma_{brec} := K \cdot \Delta_{concrec} \qquad \sigma_{brou} := K \cdot \Delta_{concrou}$

$\sigma_{brec} = 2.057$ ksi is less than $\sigma_{brou} = 2.103$ ksi O.K.

Determine the maximum flexural stress in the dowel

$x := 0, .1 .. \frac{L_{rec}}{2}$

$F_{1rec}(x) := \cosh(\beta_{rec} \cdot x) \cdot \cos(\beta_{rec} \cdot x)$

$F_{2rec}(x) := \cosh(\beta_{rec} \cdot x) \cdot \sin(\beta_{rec} \cdot x) + \sinh(\beta_{rec} \cdot x) \cdot \cos(\beta_{rec} \cdot x)$

$F_{3rec}(x) := \sinh(\beta_{rec} \cdot x) \cdot \sin(\beta_{rec} \cdot x)$

$F_{4rec}(x) := \cosh(\beta_{rec} \cdot x) \cdot \sin(\beta_{rec} \cdot x) - \sinh(\beta_{rec} \cdot x) \cdot \cos(\beta_{rec} \cdot x)$

$$\theta_{aprec} := \frac{P_{rec}}{2 \cdot E \cdot I_{rec} \cdot (\beta_{rec})^2} \cdot \frac{(C_{2rec})^2 - 2 \cdot C_{3rec} \cdot C_{1rec}}{C_{11rec}}$$

$$\theta_{amrec} = \frac{P_{rec} \cdot \frac{z}{2}}{E \cdot I_{rec} \cdot \beta_{rec}} \cdot \frac{C_{3rec} \cdot C_{4rec} + C_{2rec} \cdot C_{1rec}}{C_{11rec}}$$

$$M_{prec}(x) = -y_{aprec} \cdot 2 \cdot E \cdot I_{rec} \cdot (\beta_{rec})^2 \cdot F_{3rec}(x) - \theta_{aprec} \cdot E \cdot I_{rec} \cdot \beta_{rec} \cdot F_{4rec}(x) - \frac{P_{rec}}{2 \cdot \beta_{rec}} \cdot F_{2rec}(x)$$

$$M_{mrec}(x) = -y_{amrec} \cdot 2 \cdot E \cdot I_{rec} \cdot (\beta_{rec})^2 \cdot F_{3rec}(x) - \theta_{amrec} \cdot E \cdot I_{rec} \cdot \beta_{rec} \cdot F_{4rec}(x) - \frac{P_{rec} \cdot z}{2} \cdot F_{1rec}(x)$$

$$F_{1rou}(x) = \cosh(\beta_{rou} \cdot x) \cdot \cos(\beta_{rou} \cdot x)$$

$$F_{2rou}(x) = \cosh(\beta_{rou} \cdot x) \cdot \sin(\beta_{rou} \cdot x) + \sinh(\beta_{rou} \cdot x) \cdot \cos(\beta_{rou} \cdot x)$$

$$F_{3rou}(x) = \sinh(\beta_{rou} \cdot x) \cdot \sin(\beta_{rou} \cdot x)$$

$$F_{4rou}(x) = \cosh(\beta_{rou} \cdot x) \cdot \sin(\beta_{rou} \cdot x) - \sinh(\beta_{rou} \cdot x) \cdot \cos(\beta_{rou} \cdot x)$$

$$\theta_{aprou} = \frac{P_{rou}}{2 \cdot E \cdot I_{rou} \cdot (\beta_{rou})^2} \cdot \frac{(C_{2rou})^2 - 2 \cdot C_{3rou} \cdot C_{1rou}}{C_{11rou}}$$

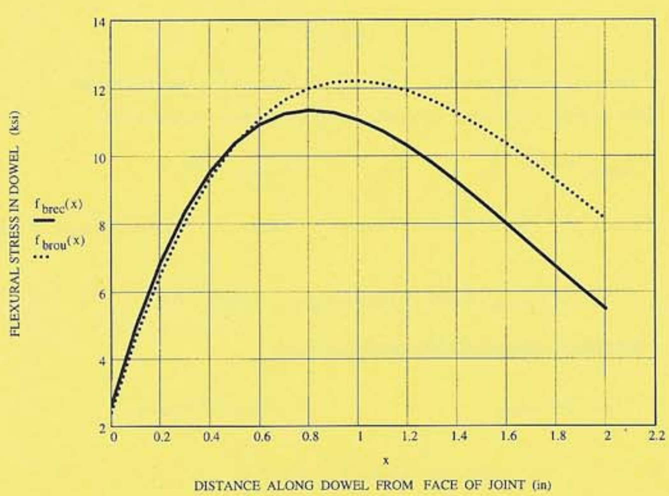
$$\theta_{amrou} = \frac{P_{rou} \cdot \frac{z}{2}}{E \cdot I_{rou} \cdot \beta_{rou}} \cdot \frac{C_{3rou} \cdot C_{4rou} + C_{2rou} \cdot C_{1rou}}{C_{11rou}}$$

$$M_{prou}(x) = -y_{aprou} \cdot 2 \cdot E \cdot I_{rou} \cdot (\beta_{rou})^2 \cdot F_{3rou}(x) - \theta_{aprou} \cdot E \cdot I_{rou} \cdot \beta_{rou} \cdot F_{4rou}(x) - \frac{P_{rou}}{2 \cdot \beta_{rou}} \cdot F_{2rou}(x)$$

$$M_{mrou}(x) = -y_{amrou} \cdot 2 \cdot E \cdot I_{rou} \cdot (\beta_{rou})^2 \cdot F_{3rou}(x) - \theta_{amrou} \cdot E \cdot I_{rou} \cdot \beta_{rou} \cdot F_{4rou}(x) - \frac{P_{rou} \cdot z}{2} \cdot F_{1rou}(x)$$

$$f_{brec}(x) = \frac{M_{prec}(x) - M_{mrec}(x)}{S_{rec}} \qquad f_{brou}(x) = \frac{-M_{prou}(x) - M_{mrou}(x)}{S_{rou}}$$

(Note: negative values for the moments are used to give positive value stresses)



From the graph above, the maximum flexural stress for the rectangular plate dowel occurs at x= 0.8" and at x=0.97" for the round dowel.

$f_{brec}(0.8) = 11.341$ ksi is less than $f_{brou}(0.97) = 12.206$ ksi O.K.

Determine the maximum shear stress in the dowel

$$f_{vrec} = \frac{P_{rec}}{A_{rec}} \qquad f_{vrou} = \frac{P_{rou}}{A_{rou}}$$

$f_{vrec} = 2.167$ ksi is less than $f_{vrou} = 2.264$ ksi O.K.

Plate Dowels for Slabs on Ground

by Wayne W. Walker and Jerry A. Holland

SUMMARY OF PERFORMANCE VALUES

	RECTANGULAR PLATE DOWELS			ROUND DOWELS		
Deflection	$\Delta_{slabrec} = 0.00593$	in	is less than	$\Delta_{slabrou} = 0.00606$	in	O.K.
Bearing	$\sigma_{brec} = 2.057$	ksi	is less than	$\sigma_{brou} = 2.103$	ksi	O.K.
Flexural	$f_{brec}(0.8) = 11.341$	ksi	is less than	$f_{brou}(0.97) = 12.206$	ksi	O.K.
Shear	$f_{vrec} = 2.167$	ksi	is less than	$f_{vrou} = 2.264$	ksi	O.K.

APPENDIX "C"

The following are the peak performance values for the diamond plate dowel with a one kip load applied at the face of the joint:

Diamond Plate Thickness	Deflection	Bearing Stress	Flexural Stress	Shear Stress
1/4"	0.003918"	1.0255 ksi	6.7641 ksi	1.2275 ksi
3/8"	0.002534"	0.7147 ksi	3.6523 ksi	0.8184 ksi
1/2"	0.001910"	0.5607 ksi	2.3753 ksi	0.6138 ksi
3/4"	0.001402"	0.4298 ksi	1.2335 ksi	0.4092 ksi

The following are the performance values for round dowels with a one kip load applied at the face of the joint:

Round Dowel Diameter	Deflection	Bearing Stress	Flexural Stress	Shear Stress
3/4"	0.00606"	2.1033 ksi	12.200 ksi	2.264 ksi
1.0"	0.00361"	1.2550 ksi	6.216 ksi	1.273 ksi
1.25"	0.00242"	0.8422 ksi	3.698 ksi	0.815 ksi

APPENDIX "D"

The following is an example problem showing how the spacing for the 1/4" diamond plate dowel plate was determined so the performance would be equivalent to the 3/4" round dowel spaced at 12". The performance values were taken from Appendix "C".

For deflection ⟶ $\frac{0.00606" \text{ X } 12"}{0.003918"} = 18.56"$ ⟵ **Controls**

For bearing ⟶ $\frac{2.1033 \text{ ksi X } 12"}{1.0255 \text{ ksi}} = 24.61"$

For flexural ⟶ $\frac{12.200 \text{ ksi X } 12"}{6.7641 \text{ ksi}} = 21.64"$

For shear ⟶ $\frac{2.264 \text{ ksi X } 12"}{1.2275 \text{ ksi}} = 22.13"$

End of Appendices